STEP METHOD FOR COMPUTING TOTAL SEDIMENT LOAD BY THE MODIFIED EINSTEIN PROCEDURE

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Introduction

The modified Einstein procedure for computing total sediment load was conceived by Messers B. R. Colby and C. H. Hembree of the Geological Survey, Lincoln, Nebraska. It is the product of several years' field research conducted jointly by the Bureau of Reclamation and the Geological Survey on wide, shallow, alluvial streams in Nebraska. The full development of the procedure is given in Geological Survey Water Supply paper number 1357 entitled "Computations of Total Sediment Discharge Niobrara River Near Cody, Nebraska." The procedure has the merit of being adaptable to standard routine field observations presently conducted by the Quality of Water division of the Geological Survey. It has greater accuracy for computing the transport of the individual sediment size fractions of all the methods presently available. Because it is adaptable to the routine Geological Survey field observations, it is the most economical method of securing a reasonably accurate determination of total load.

The procedure, as the title indicates, is a modification of "The Bed-Load Function for Sediment Transportation in Open Channel Flows" by Hans Albert Einstein and published by the Department of Agriculture as technical bulletin 1026, Sept. 1950. The symbols and formulas are those used by Einstein in his paper with modification to fit the new procedure. To understand the basic principles of the modified procedure a study should be made of Einstein's original paper.

This step method and the accompanying form is presented to standardize and document computations of the modified procedure. It will also make it possible for personnel to make the computations without thoroughly understanding all the theory of the various functions.

Sample Procedure

Figures of base data are first entered on the computation form in the box headed "Data." The water discharge, width, mean velocity, average depth, and average depth d_s at the sampling verticals are all in pound-foot-second units. These units are used throughout the computations except for sediment discharges that are represented by symbols with a Q and are in tons per day. In the example the width, water discharge, mean velocity, average depth, and average depth d_s are 230, 113, 2.08, 0.98, and 1.22, respectively. This information may be obtained from the water discharge measurement and sediment sampling field notes. It may be found for most cases that d and d_s are very nearly the same and the d may be substituted for d_s. This is particularly true if the equal transit method of suspended sampling is used. Additional basic data that may be obtained from the water discharge and sediment sampling notes are the cross-sectional area, the water temperature, and the type sampler used. The water temperature is sometimes overlooked by the field observer and the mean air temperature for the day from the nearest Weather Bureau station can be substituted. In the example the area, water temperature, and type of sampler used were 111 sq. ft. 64° F and hand sampler--DH-48, respectively.

The analysis report of the suspended sediment samples will give the following information:

1. The mean concentration of the suspended sediments in parts per million (ppm)
2. The size distribution of the suspended samples
3. The sampled suspended load transported in tons per day computed from

$$Q_{sm} = Q_w \times \text{Conc} \times .0027$$

In the example the concentration is 262 ppm and the load measured is 163 tons per day.

Additional field data that must be obtained are adequate samples of the bed material. The term adequate is at this time not well defined. Present minimum standards are samples obtained at the third points of discharge. Where a large number of samples have been obtained at different times for a particular section the average of the results appear satisfactory. Do not use dry sand bar samples or those obtained from a dry bed after the water has receded.

Analysis of the bed material samples will be reported as size distribution in percent finer. (Note fraction breakdown on form, Column 1.) i_b in Column 7 of the form is the percent of bed material within the size brackets and can be computed from the size distribution of the bed material. In the data Column D_{65} (K_s) and D_{35} is the size in feet of the bed material that is 65 percent finer and 35 percent finer, respectively. These sizes are determined from a logarithmic probability graph of the bed material size distribution as shown in Plate 1 of this example. Values of 0.32 mm = 0.00105 feet, and 0.23 mm = 0.00075 feet for D_{65} (K_s) and D_{35}, respectively, are read from Plate 1 (1 mm = 0.003281 feet).

The water temperature is 64° F in the example. The kinematic viscosity (ν) is 1.14×10^{-5} ft^2/sec as obtained from Plate 2. This figure is entered in the appropriate space in the data column.

The computations begin with the determination of (SR) from the following equation wherein $\overline{u}$ equals 2.08, K_s equals 0.00105, and d equals 0.98.

$$\sqrt{SR} = \frac{\bar{u}}{32.63 \log_{10}\left[\frac{12.27\,(x)\,(d)}{K_s}\right]}$$

In the above equation assume on the basis of past experience or an approximate computation on scratch paper that x equals 1.54. Then

$$\sqrt{SR} = \frac{208}{32.63 \log_{10}\left[\frac{(12.27)\,(1.54)\,(0.98)}{0.00105}\right]} = 0.0105$$

$$(SR) = 0.000225$$

The shear velocity (u'_*) is computed from $(u'_*) = \sqrt{SR}\ \sqrt{g}$ wherein g is the acceleration due to gravity and is equal to 32.2. Then

$$u'_* = (0.0150)\,(5.68) = 0.0853$$

The kinematic viscosity, ν, is 1.14×10^{-5} ft^2/sec. The thickness of the laminar sublayer, δ, is

$$\delta = \frac{11.6\nu}{u'_*}$$

Then

$$\delta = \frac{(11.6)\,(1.14)\,(10^{-5})}{0.0853} = 0.00155$$

so

$$\frac{K_s}{\delta} = \frac{0.00105}{0.00155} = 0.68$$

and from Plate 3, x = 1.54. As the assumed x is the same as the computed x, no recomputation is necessary. In fact, the whole quantity under the log sign is so large that x can differ considerably from its assumed numerical value without necessitating a recomputation.

By definition

$$P = 2.303 \log_{10}\left[\frac{30.2\,(x)\,(d)}{K_s}\right]$$

Then

$$P = 2.303 \log_{10}\left[\frac{(30.2)\,(1.54)\,(0.98)}{0.00105}\right] = 10.7$$

A' is equal to d_n/d_s wherein d_n is the vertical distance, in feet, not sampled; that is, the distance from the bottom of the sampled zone to the streambed which depends upon the type sampler used, and d_s is the mean depth of the water at the sampled verticals.

$$A' = \frac{0.3}{1.22} = 0.246$$

From Plate 4 for a P of 10.7 and the fraction of the depth not sampled (A') it can be determined that 80 percent of the streamflow was sampled. The sediment discharge through the sampled zone Q'_s then can be computed as 80 percent of the Q_{sm}

$$Q'_s = .80 \times 163 = 130 \text{ tons per day.}$$

This figure represents the total for Column 12 on the form.

The next major step is the computation of $i_B Q_B$. Column 1 contains the size breakdown of the sediment fractions. It may be noted there is a break in the size consideration below .0625 mm. This has purposely been done to allow versatility in use of the form. The data for the example has no suspended material finer than 0.002 mm. If the problem being solved has finer suspended material the top two lines may be used and the third line of computations omitted. Column 2 is the geometric mean in millimeters (square root of the products) of the size fraction. Column 3 is the geometric mean of the size fraction in feet.

The intensity of shear on the particles, ψ, is computed from the following equations:

$$\psi = \frac{1.65\ D35}{(SR)}$$

$$\psi = \frac{0.66\ D}{(SR)}$$

in which D is the geometric mean particle size from Column 3. The number 1.65 is the specific gravity of the sediment particles minus the specific gravity of water. The larger ψ from the above equations is listed in Column 4 for each size range in which the bed transport is appreciable. It may be noted the i_b Column does not total 100 percent. In the example the finer or coarser sizes than those used would not have appreciable bed load ($i_B Q_B$) discharges. It may be necessary in other problems to compute a different range of sizes.

From Plate 5 the intensity of bed-load transport $\phi_*/2$ (note that ϕ_* must be converted to $\phi_*/2$) is determined for each ψ and is listed in Column 5.

The equation for computing $i_B Q_B$ is

$$\frac{i_B Q_B}{43.2\ W} = 1,200\ D^{3/2}\ i_b \phi_*/2$$

also

$$i_B q_B = \frac{iBQB}{43.2\ W}$$

Column 6 contains the numerical values of $1,200\ D^{3/2}$ for the different geometrical mean particle sizes. Column 7 contains the fraction, i_b, of the bed material in each size range. In Column 8 is listed the bed-load discharge, in pounds per second per foot of width, as computed from

$$i_B q_B = 1,200\ D^{3/2}\ i_b\ \phi_*/2$$

Each of the figures from Column 8 is then multiplied by the width of the section, 113 feet, and by 43.2, the factor to convert sediment discharge in pounds per second to tons per day. Column 9 is the product of 43.2 w. Column 10 is the product of $i_B q_B$ and 43.2 w.

Column 11, the percentage of the suspended material (Q'_s) in the various size fractions, is computed from the size distribution analysis of the suspended material. Plate 6 is the size distribution curve plotted on log-probability paper of the suspended sediments in the example. If the size distribution is reported in odd fractions the desired fractions may be computed from the log-probability plot. The computer should examine the suspended size distribution plotting for distortion. If the size distribution is greatly different than what should be expected in comparison with other samples at the section the answer will be greatly affected and may be questioned. Odd size analysis is not unusual and may be due to a poor size distribution analysis or just a happenstance in the sampling.

The figures in Column 12 for each size fraction are the amount of sampled transport and are computed by multiplying the percentages in Column 11 by the total transport in the sampled zone previously computed as 130 tons per day.

Column 13 contains the multipliers or ratios from Plate 7 for a temperature of 64° F. These multipliers are used to compute Z's for other size ranges from the trial-and-error Z' for the reference size range. The multipliers are merely the ratios of the 0.7 powers of the fall velocities with reference size. Fall velocities used were those determined by Rubey. In the example the reference size has been chosen as 0.00058 feet (0.125 - 0.25 mm). It should represent one of the suspended size fractions containing a substantial amount of the total sampled load. This is desirable because there is less chance for large effective error in the size distribution analysis. If another reference size fraction is used than presented in the example the multipliers may be computed by dividing the multipliers given by the multiplier for the chosen reference size. For example if 0.00116 feet (0.25-.50 mm) particle size had been chosen for the reference size then the multiplier for the 0.00058 feet particle at 64° F would be 1/1.76 or 0.568.

The figures in Column 15 are computed from

$$A'' = 2\,D/d$$

wherein D is the geometric mean size in feet and d is the mean depth of the channel cross-section.

The next step in the computation is to compute Z' by trial-and-error for the reference size range which in the example is 0.125 to 0.250 millimeters (geometric mean size, 0.00058 foot). This size range is that selected for all computations for the Niobrara River near Cody. For the reference size range, Z' is computed from the following equation:

$$\frac{Q'_s}{i_B Q_B} = \frac{I_1''}{J_1''} (PJ_1' + J_2')$$

Wherein Q'_s and $i_B Q_B$ are the amounts computed for the reference size range. The ratio of Q'_s to $i_B Q_B$ is 55/7.96 = 6.91. From Plate 8 Z' would be about 0.80. If Z' equals 0.80 and A'' equals 0.00118 I_1'' and J_1'' are respectively about 2.50 and 3.00, from Plates 9 and 10. J_1' and J_2' from Plates 10 and 11 are about 0.62 and 0.52. An A' of 0.246 as previously computed is used for entering Plates 10 and 11 for selecting J_1' and J_2'. In utilizing the "J" curves it should be noted that "J_2" is always a negative value. Note that Plate 10 serves a double purpose for selecting J_1' and J_1''. Substitution in the right hand side of the above equation gives

$$\frac{2.50}{3.00} \left[(10.7)\,(0.62) - 0.52\right] = 5.10$$

5.10 does not equate with 6.91 previously computed as the value of $Q'_s/i_B Q_B$. The slope of the line in Plate 8 indicates that the difference between 6.91 and 5.10 requires a change in Z' of about 0.04, therefor consider a Z' of 0.76. Computations on the right hand side of the equation are as follows

$$\frac{2.92}{2.60} \left[(10.7)\,(0.62) - 0.52\right] = 6.87$$

A Z' of 0.76 is satisfactory as a change in Z' of 0.01 will change the above computation about 0.30. The final computation is entered in the appropriate space in the upper right hand side of the form. Z' for the reference size 0.00058 feet is entered in Column 14. The other Z's in Column 14 are computed from the multipliers in Column 13.

Now that A', A'', and $\mathfrak{Z}'$ are known for all the size ranges, the different numerical values of I and J can be taken from Plates 9 to 12. The I_2'', J_2' and J_2'' values are always negative and are so designated on the computation form as a reminder. The minus in the form directs that they should be used as minus values. These values are tabulated in the proper Columns 16 to 19 and 21 and 22. Column 20 contains the ratio $(PJ_1'' + J_2'')/(PJ_1' + J_2')$ for each range of particle sizes. These ratios are computed from $P = 10.7$ and from entries in Columns 16 to 19. Column 23 contains the numerical values of $PI_1'' + I_2'' + 1$.

Total discharge of sediment through the cross section is next computed for entry in Column 24 by multiplying together figures from Column 12 and ratios from Column 20 for the ranges of fine particle sizes and figures from Columns 10 and 23 for the ranges of coarser particle sizes. The sum of the figures in Column 24 is the computed total sediment discharge at the section.

The question might be raised as to why the computation methods are different for the ranges of the finer particle sizes than for the ranges of the coarser particles. In the reference size range the two methods will compute the same sediment discharge if $\mathfrak{Z}'$ is precisely correct and if $i_B Q_B$ is added to the computed discharge of the finer particles. (In the sample computation, $i_B Q_B$ is not added to the computed discharge of sediment for the two ranges of smallest particle sizes because it is negligibily small.) Theoretically, either the $(PJ_1'' + J_2'')/(PJ_1' + J_2')$ or the $PI_1'' + I_2'' + 1$ method can be used throughout the range of particle sizes. Practically, the first method is limited to ranges of particle sizes for which Q'_s can be determined with fair accuracy; the second method, to ranges of particle sizes for which i_b can be determined with reasonable accuracy. Another practical limitation on the choice of method is that a given percentage of variation in $\mathfrak{Z}'$ changes the computed sediment discharges more by the first method when $\mathfrak{Z}'$ is large and more by the second method when $\mathfrak{Z}'$ is small.

Glossary of Terms

A	Stream cross-sectional area in square feet
A'	Distance of lower limit of integration above the sampling depth, as defined by the type sampler used, divided by d_s, the average depth of the water at the sampled verticals
A''	Distance of lower limit of integration above streambed divided by d, the depth of the stream (two times mean given diameter in any size group divided by the mean depth of the cross section)
Conc.	Concentration of suspended sediments in parts per million by weight
D	Geometric mean diameter of a size range
D_{35}	Particle size at which 35 percent of the bed material by weight is finer
D_{65}	Particle size at which 65 percent of the bed material by weight is finer
d	Mean depth of cross-section obtained by dividing the area by the width (d assumed equal to R in wide shallow channels)
d_n	The distance between the bottom of the sampled zone and the streambed as defined by the sampler used
d_s	The average of the total depths recorded at the sampling verticals
g	The gravity constant, 32.2 feet per second per second
I_1'	Mathematical abbreviation which contains J_1' associated with the sampling depth
I_1''	Mathematical abbreviation which contains J_1'' that is associated with the total depth through which suspended sediment is discharged
I_2'	Mathematical abbreviation which contains J_2' associated with the sampling depth
I_2''	Mathematical abbreviation which contains J_2'' that is associated with the total depth through which suspended sediment is discharged (I_2'' is always negative)
i_BQ_B	Sediment discharge through the bed layer of particles of a size class, in tons per day

$i_B q_B$	Sediment discharge through the bed layer of particles of a size class, in pounds per second per foot of width
i_b	Fraction by weight of bed material in a size range
J_1'	Equals $\int_{A'}^{1} \left(\frac{1-y}{y}\right)^{Z'} dy$
J_1''	Equals $\int_{A''}^{1} \left(\frac{1-y}{y}\right)^{Z'} dy$
J_2'	Equals $\int_{A'}^{1} \left(\frac{1-y}{y}\right)^{Z'} \log_e (y)\, dy$
J_2''	Equals $\int_{A''}^{1} \left(\frac{1-y}{y}\right)^{Z'} \log_e (y)\, dy$

NOTE: J_2' and J_2'' are always negative

K_s	Roughness diameter, that particle size of bed material for which 65 percent by weight is finer
P	Equals $2.303 \log_{10} \left[\frac{30.2\,(x)\,(d)}{K_s}\right]$
Q	Water discharge in cubic feet per second
Q_{sm}	Sediment discharge measured by sampling in tons per day
Q'_s	Sediment discharge in sampling zone of a size range in tons per day
R	Hydraulic radius
S	Slope of the energy gradient
T	Water temperature, in degrees Fahrenheit
$\bar{u}$	Average velocity of flow in feet per second
$\bar{u}_*$	Shear velocity equal to $\sqrt{RSg}$
V_s	Fall velocity of sediment particles (Rubey)
w	Width of the stream channel
X	Dimensionless parameter determined from Plate 3

y	Distance above the streambed
z'	Exponent for vertical distribution of sediment
δ	Thickness of laminar sublayer
ν	Kinematic viscosity
ϕ_*	Intensity of bed-load transport
ψ	Function for correlating effect of flow with intensity of sediment transport

PLATE I

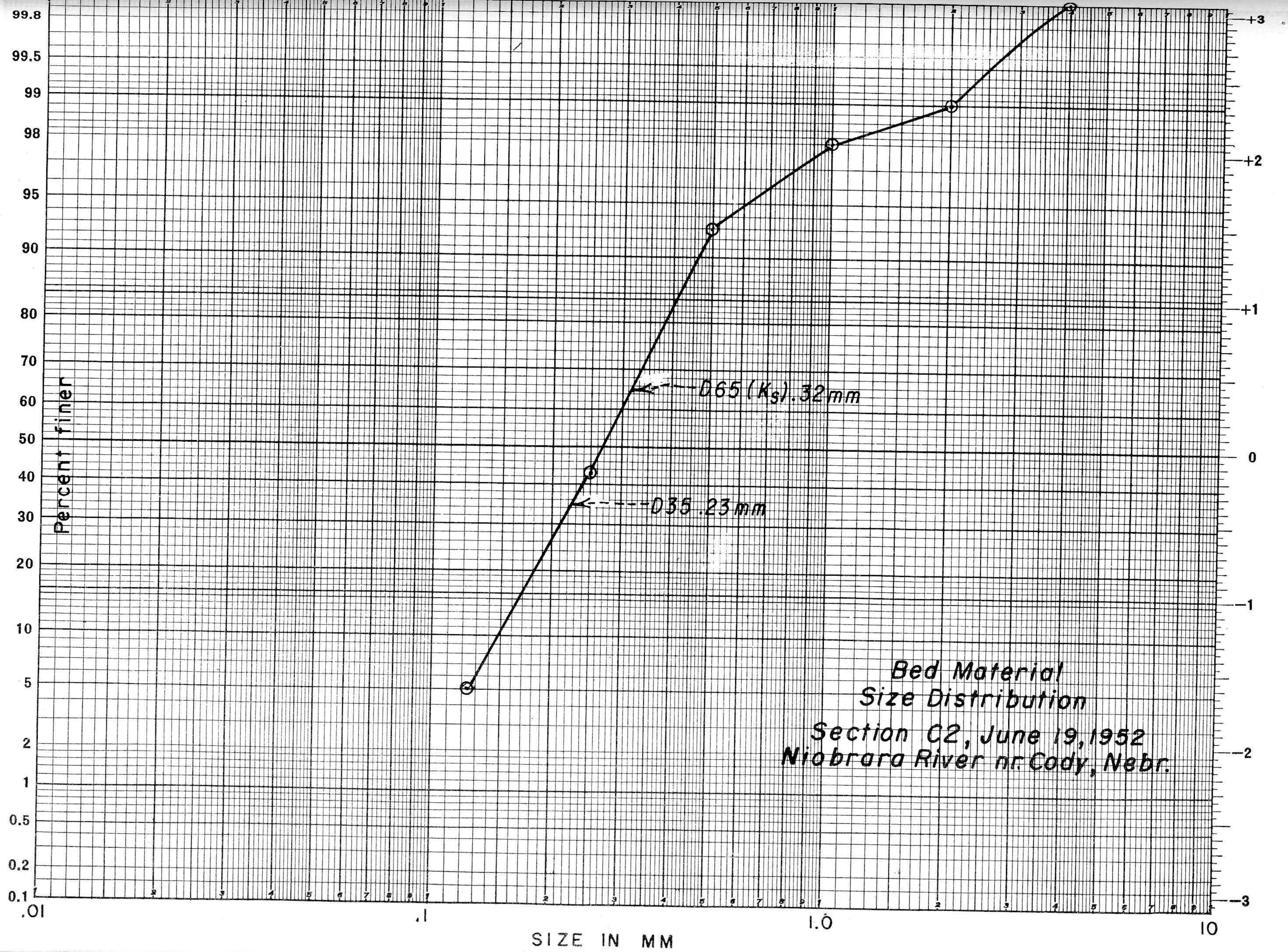

Bed Material
Size Distribution
Section C2, June 19, 1952
Niobrara River nr. Cody, Nebr.

PLATE 2

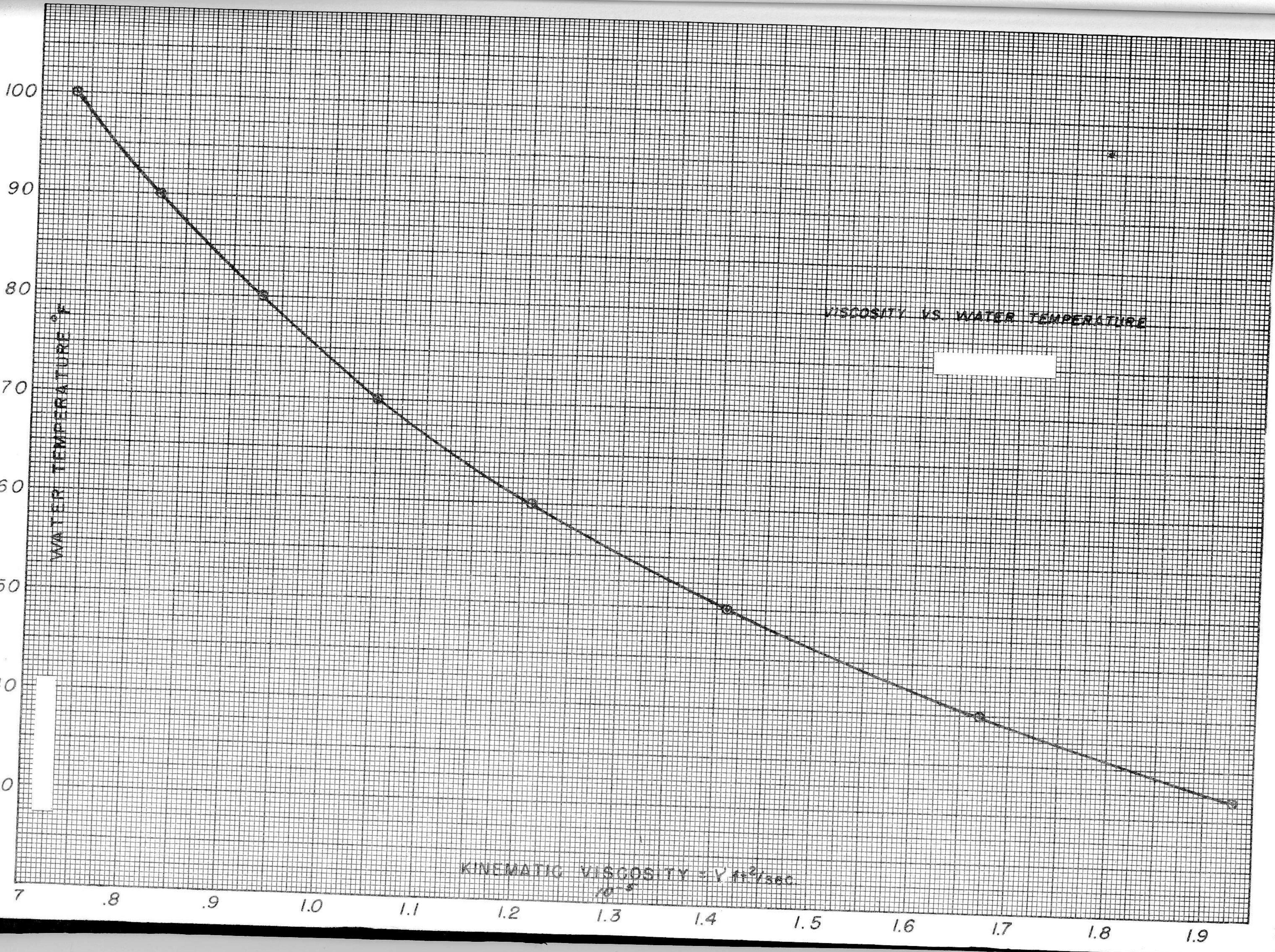
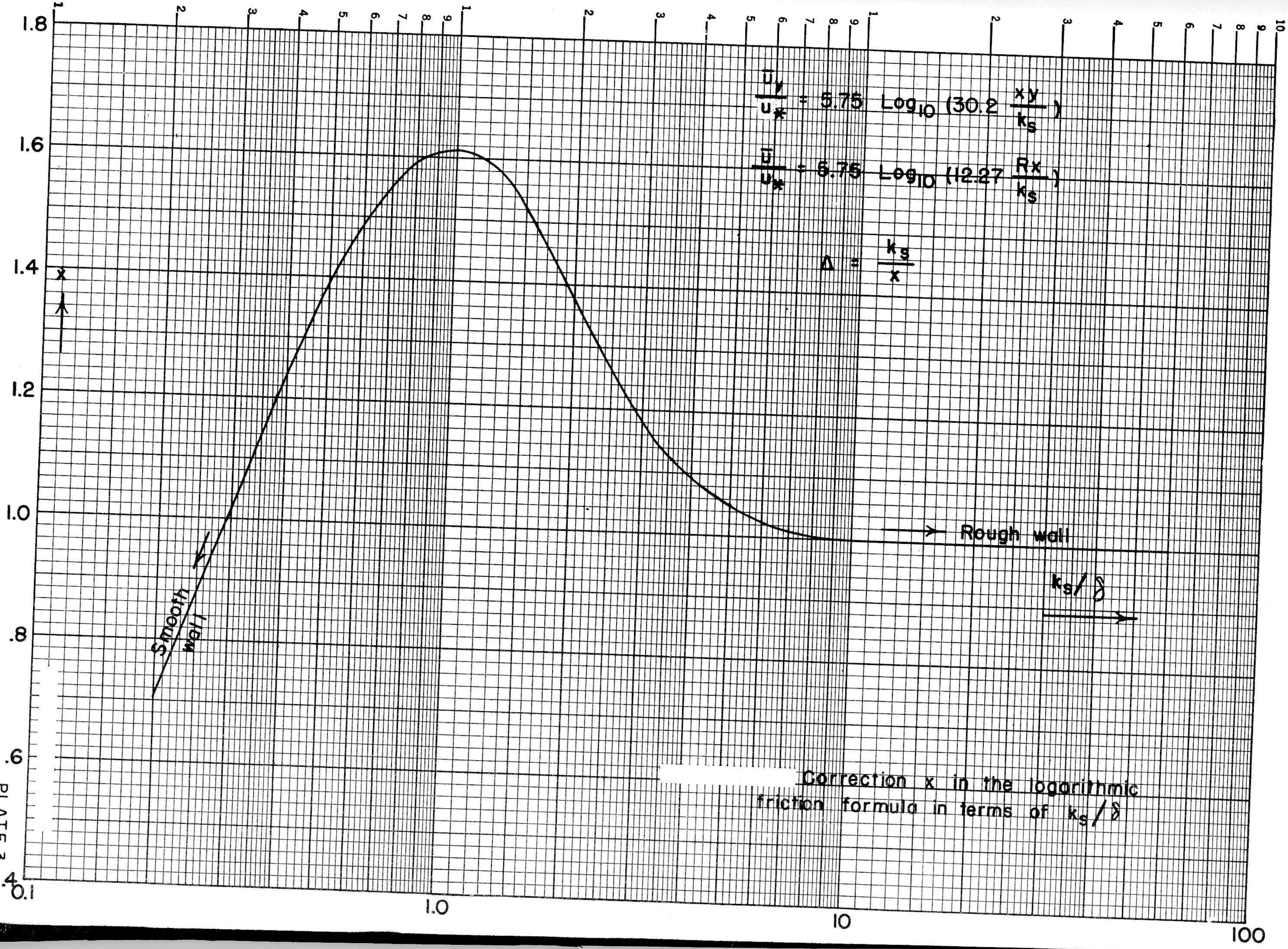

VISCOSITY VS. WATER TEMPERATURE
WATER TEMPERATURE °F
100
90
80
70
60
50
40
30
KINEMATIC VISCOSITY = ν ft²/sec. 10⁻⁵
7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9

PLATE 3



Correction x in the logarithmic friction formula in terms of k_s / δ

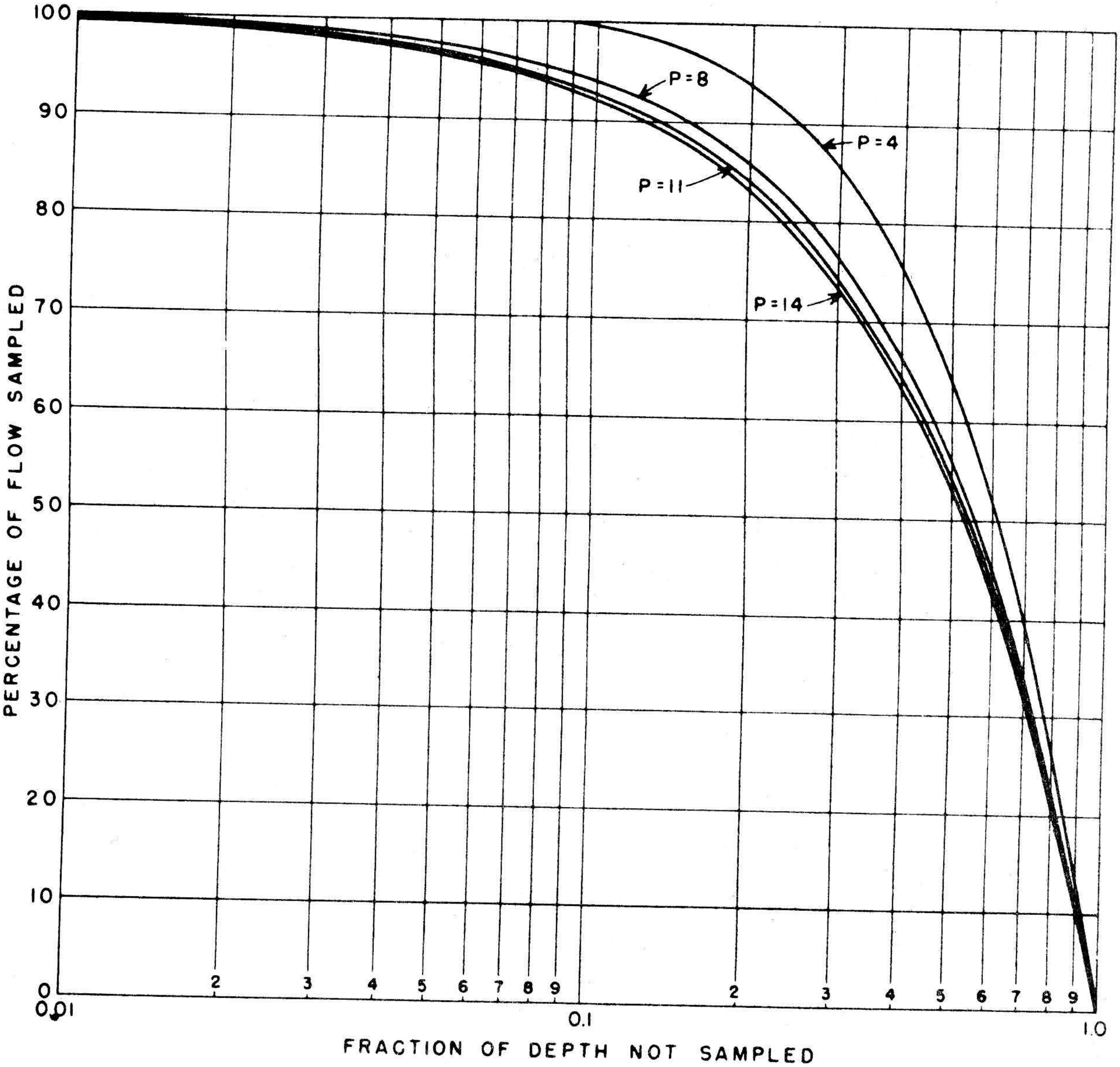
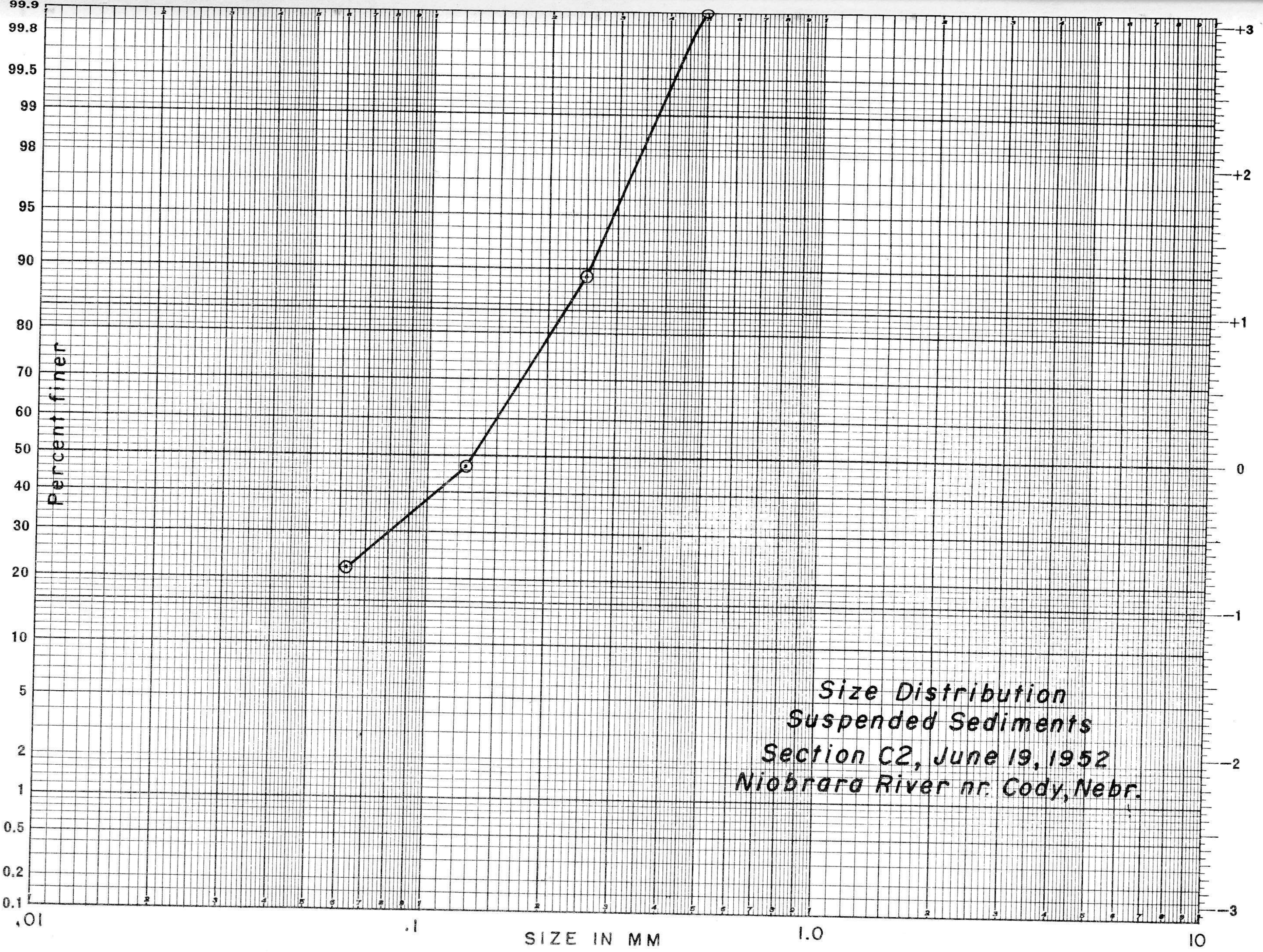
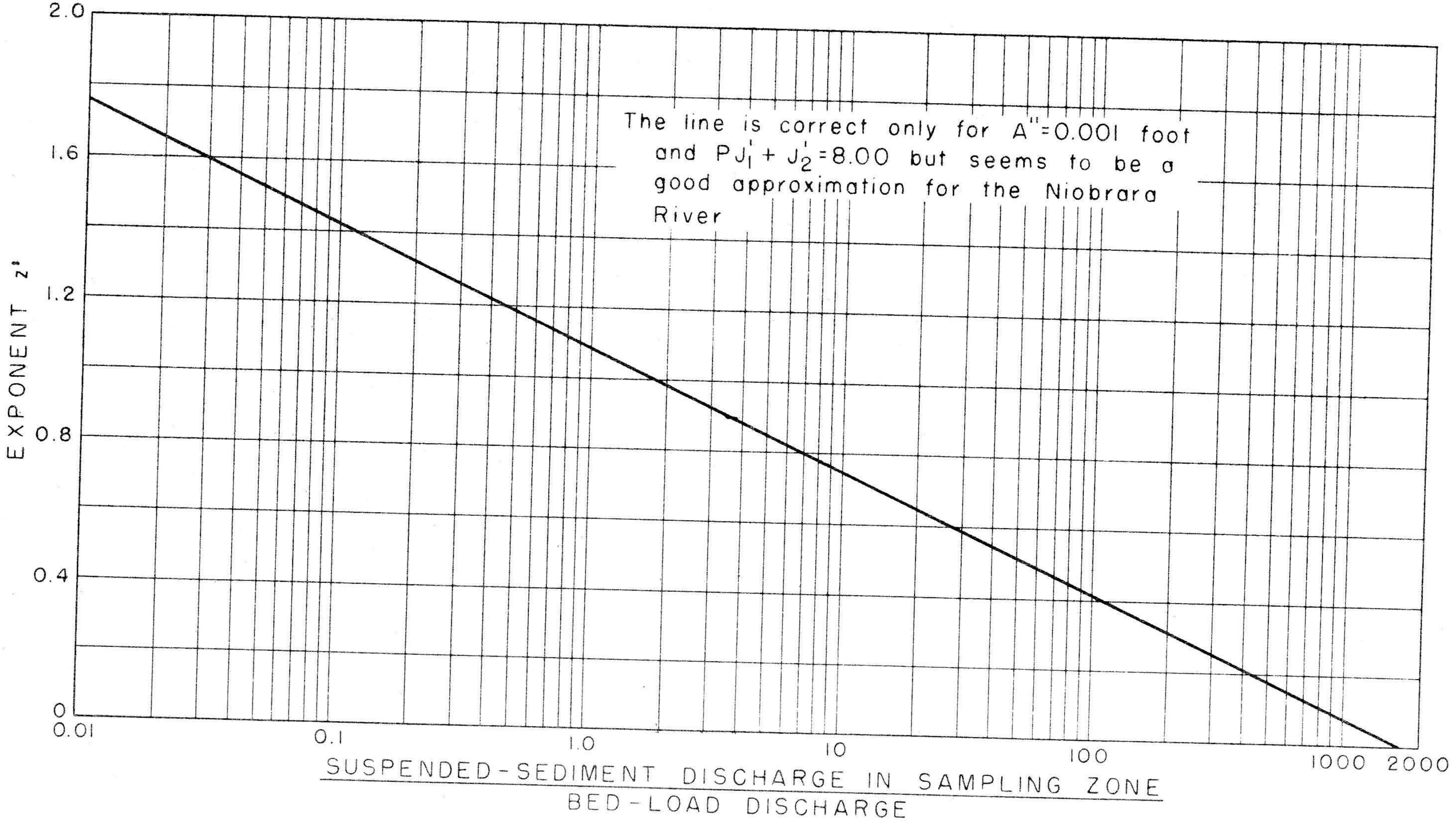


Plate 4 --Vertical distribution of streamflow.

PLATE 6



Size Distribution
Suspended Sediments
Section C2, June 19, 1952
Niobrara River nr. Cody, Nebr.



The line is correct only for $A'' = 0.001$ foot and $PJ_1' + J_2' = 8.00$ but seems to be a good approximation for the Niobrara River

Plate 8 Approximate relation of z' to the ratio of suspended-sediment discharge in the sampling zone to bed-load discharge.